# Modeling the fluid dynamics and molecular dynamics

What is 'Fluid Flow'?

When one applies pressure on a solid

- Structure deformation => energy conservation, elastic

When one applies pressure on a fluid

- Molecular collisions => momentum transfer, dissipation.





#### The number of water molecules in $(10 \text{ nm})^3$ volume ? How about in $(1 \mu \text{m})^3$ ? Challenges of MD --

Simulation of methane in ice in 10x10x10 nm<sup>3</sup> volume for 2  $\mu$ s

Walsh, Sum, Wu, Science (2009)



How to model the dynamics of macromolecules in a solution ? Coarse-grain : Group many water molecules as an effective particle <u>Dissipative particle dynamics</u>, <u>Multi-particle collision dynamics</u>

Continuum : Solve the Navier-Stokes equation by <u>Finite Volume</u>, <u>Finite</u> <u>Element</u>, <u>Boundary integral</u>

Pseudo-continuum: Lattice Boltzmann

**Cost ~O(# of elements)** 

### **Continuum Fluid Modeling**

Particle motion perturbs and contributes to the overall velocity field

$$\Delta \vec{\mathbf{v}}(\vec{r}, \vec{r}_0, \vec{f}_0) = \Delta \vec{\mathbf{v}}_s(\vec{r} - \vec{r}_0) + \Delta \vec{\mathbf{v}}_W(\vec{r}, \vec{r}_0) = \mathbf{\Omega}(\vec{r}, \vec{r}_0) \cdot \vec{f}_0$$
Free space Wall correction  $\sqrt{\mathbf{v}}$  Force  $\mathbf{S}$  tokes Flow
$$0 = -\nabla p + \eta \nabla^2 \vec{\mathbf{v}}_W$$

$$0 = \nabla \cdot \vec{\mathbf{v}}_W$$
Solved w/
Finite Element Method
HI is dependent on particle positions as a function of time
$$\mathbf{Wall}$$



# **Multi-Scale Modeling Approach**



2 nm

Capture essential physics and optimize computational complexity Flow timescale ~ minutes => coarse-grained model

# **Coarse-grained DNA Molecule**

DNA molecule - short length scale: double helix long length scale: flexible polymer

Model parameters matched to material specific properties of TOTO-1 stained  $\lambda$ -DNA

Approximate DNA as a *Worm-like Chain* 

• Molecular scale features are coarse-grained

• Blobs interact with each other through Elastic spring 'Soft' excluded volume Hydrodynamic interactions contour length ~  $o(10-10^2) \mu m$ radius of gyration ~  $o(1) \mu m$ 

 $\xi_p\approx 50~nm$ 



# **Brownian Dynamics and Hydrodynamics**



$$d\vec{R} = \left[\vec{U} + \frac{1}{k_B T} \mathbf{D} \cdot \vec{F} + \nabla \cdot \mathbf{D}\right] dt + \sqrt{2} \mathbf{B} \cdot d\vec{W}$$
  
Flow Field Force Drift Brownian

Solve Continuum Hydrodynamics

Diffusion tensor is a function of the *instantaneous molecular configuration* and *geometry* of the device

$$\mathbf{D}_{ij} = \frac{k_B T}{\zeta} (\mathbf{I} \delta_{ij} + \zeta \Omega_{ij}) \qquad \mathbf{Cost} \sim \mathbf{O}(N^{2.2})$$

 $\zeta$ : particle friction coef.

 $\mathbf{\Omega}_{ij}$ : hydrodynamic interaction tensor

#### Thermal Brownian Motion

Fluctuation-dissipation theorem couples hydrodynamics to Brownian forces

$$\sum_{j=1}^{N_h} \mathbf{B}_{ij} \mathbf{B}_{kj} = \mathbf{D}_{ik}$$

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#### **DNA in microflow : Flow-induced DNA migration**



Chain stretch when *flow shear rate > (relaxation time )*<sup>-1</sup> Steady state is reached after 100 s

Chen et al., Macromolecules (2007)

DNA distribution in microchannel





# **DNA Separation in Microcapillary**



λ-DNA in microcapillary flowSugarman & Prud'homme (1988)Chen et al.(2005)

Detection points at 25 cm and 200 cm

$$R_f = \frac{\text{avg. DNA velocity}}{\text{max. fluid velocity}}$$

Longer DNA → higher velocity Separation by MW is possible

Key challenges : resolution and speed **Cost** ~  $O(N^{2.2})$ 

#### T2 DNA after 100 s oscillatory Poiseuille flow



# **Modeling Cell Dynamics in Microflow**

Flow separation of soft particles / droplets by size, elasticity, shape Mechanisms : Deformation – flow interactions, fluid inertia



**Elasticity** 



Size



Hsu and Chen, J. Chem. Phys., (2010); YL Chen, RSC Advances (2014)

### **Multi-Particle Collision**



Fluid hydrodynamics by simulating fluid particle dynamics in two steps:

Streaming – Newton EOM  $\mathbf{r}_i(t + \Delta t) = \mathbf{r}_i(t) + \Delta t \mathbf{v}_i(t),$ 

Collision – thermalize fluid velocities after collisions

 $\mathbf{v}_i(t + \Delta t) = \mathbf{u}(t) + \mathbf{v}_{i,\text{ran}} - \sum_{\text{cell}} \mathbf{v}_{i,\text{ran}} / N_c$  thermostat

Angular momentum conservation

$$+ \left\{ m\Pi^{-1} \sum_{j \in \text{cell}} \left[ \mathbf{r}_{j,\text{c}} \times (\mathbf{v}_j - \mathbf{v}_j^{\text{ran}}) \right] \times \mathbf{r}_{i,\text{c}} \right\}$$

$$\mathbf{u} = \frac{m\sum_{i=1}^{N_{\mathrm{c}}} \mathbf{v}_{i} + m_{\mathrm{m}} \sum_{i=1}^{N_{\mathrm{m}}} \mathbf{w}_{i}}{N_{\mathrm{c}}m + N_{\mathrm{m}}m_{\mathrm{m}}},$$

Momentum exchange between fluid particle and monomers

Gompper et al., Adv. Polymer Sci. 221 (2009)

### **MPC modeling of RBC dynamics**

MPC has been applied to characterized the shape change and dynamic change of elastic particles with different membrane viscosity and flow rate



Gompper et al., Adv. Polymer Sci. 221 (2009)

#### **Boundary Integral Method**

Solving for the flow field due to a cell with a boundary integral representation





Stokes flow of point sources

$$u_{j}^{D}(\mathbf{x}_{0}) = -\frac{1}{8\pi\mu} \iint_{P^{+},C,B,T} G_{ij}(\mathbf{x},\mathbf{x}_{0}) f_{i}^{D}(\mathbf{x}) \,\mathrm{d}S(\mathbf{x}) + \frac{1}{8\pi} \iint_{P,C,B,T} u_{i}^{D}(\mathbf{x}) T_{ijk}(\mathbf{x},\mathbf{x}_{0}) n_{k}(\mathbf{x}) \,\mathrm{d}S(\mathbf{x}),$$

Stokes flow of point dipoles + sources

$$G_{ij}(\mathbf{x}, \mathbf{x}_0) = \frac{\delta_{ij}}{|\mathbf{x} - \mathbf{x}_0|} + \frac{(x_i - x_{0_i})(x_j - x_{0_j})}{|\mathbf{x} - \mathbf{x}_0|^3}$$

$$T_{ijk}(\mathbf{x}, \mathbf{x}_0) = -6 \frac{(x_i - x_{0_i})(x_j - x_{0_j})(x_k - x_{0_k})}{|\mathbf{x} - \mathbf{x}_0|^5}$$

Barthes-Biesel et al., J. Comp. Phys (1989) Pozrikidis, Modeling and Simulation of Capsules and Biological Cells (2003)

### **Application of Immersed Boundary Method**



The immersed boundary method has been applied to the flow of o(100) vesicles.

Qualitatively captures the Fahraeus-Lindqvist effect.

Limitation : Cells do not come too close to each other in the computation. Near-field hydrodynamics require very fine fluid grid.

Doddi & Bagchi, (2009) Pan, Shi, Glowski (2010)

# The Boltzmann equation and lattice Boltzmann

Duenweg and Ladd, Adv. In Polymer Sci (2009)

Replace continuum fluid with discrete fluid positions  $x_i$  and discrete velocity  $c_i$ 

 $n_i(r,v,t) =$  fluid velocity distribution function

 $n_i(r+c_i\Delta t,t+\Delta t) = n_i(r,t) + \Delta_i[\boldsymbol{n}(r,t)]$ 

#### n function momer distribu

#### 3D, 19-vector model

Hydrodynamic fields are moments of the velocity distribution function

$$\rho = \sum_{i} n_i \quad j = \sum_{i} n_i c_i$$

 $\Pi = \sum_{i} n_i c_i c_i$ 

**Ghost Moments** 

Boltzmann eqn.

$$\partial_t n + v \cdot \nabla n = \left(\frac{dn}{dt}\right)_{coll}$$

$$\Delta_i[\boldsymbol{n}(r,t)] = \sum_j L_{ij}(n_j - n_j^{eq})$$

 $L_{ij} =$ local collision operator

 $=1/\tau$  in the simplest approx.

# Local collisions and propagation

lbe.c

- Hydrodynamics comes from local collisions between fluid particles
- Collisions are uncorrelated between timesteps



$$n_i(r+c_i,t+1) = n_i(r,t) + L_{ij}[n_j(r,t) - n_j^{eq}(r,t)]$$

Common 3D model has 19 velocities  $c_i$ : 0,  $\pm x$ ,  $\pm y$ ,  $\pm z$ ,  $\pm xz$ ,  $\pm xy$ ,  $\pm yz$ 

#### **Simulation Steps**

- Hydrodynamics comes from local collisions at solid-fluid boundaries
- Collisions are uncorrelated between timesteps

$$n_i(r+c_i,t+1) = n_i(r,t) + L_{ij}[n_j(r,t) - n_j^{eq}(r,t)]$$



- 1. Equilibration of fluid velocity u<sub>i</sub>
- Fluid particle collision instant exchange of mass and momentum
- 3. Fluid-solid boundary condition
- 4. Propagation of fluid velocity distribution functions

#### **Solid-Fluid Boundary Conditions**



Fluid-Particle Boundary

bnodes.c



Nguyen & Ladd (2002)

Momentum is transferred across the fluid-particle boundary

#### Solid-Fluid Boundaries

#### bnodes.c

# The Bounce Back Rule No slip - Fluid velocity at wall is zero Fluid site $n_{i'}(r,t+1) = n_i(r,t_+)$ Stationary Wall Wall site $n_i(r+c_i,t+1) = n_{i'}(r+c_i,t_+)$

 $n_i(r,t_+)$  = postcollision distribution of velocity i at r

#### Pressure-driven flow between parallel plates

Ux

Parabolic velocity profile attained by applying no-slip and constant pressure (momentum flux) to the fluid



Lattice #

 $\nabla p = \eta \nabla^2 \mathbf{u} + \mathbf{f}$ 

#### Creeping flow around a fixed particle



V=0.005

#### **Fluid Fluctuations**

Brownian motion particularly important for submicron particles

Noises are present for all non-conserved eigenmodes of the DdQq system How to add proper noises that satisfly the fluctuation-dissipation theorem ?

Reference: Adhikari, Stratford, Cates, Wagner (2004)

- 1. Find eigenvectors through Gram-Schmidt orthogonalization
- 2. Find variance of the non-conserved stress and ghost modes
- 3. Apply FDT and generate noise for the  $c_i$

With assumptions : correlations are **k**-independent

stationarity of equal time-correlators

lbe.c

From FDT: 
$$\left\langle n'^{a}n'^{b}\right\rangle = \frac{\tau_{a} + \tau_{b} - 1}{\tau_{a}\tau_{b}} \left\langle \delta M^{a} \delta M^{b} \right\rangle$$

 $\tau_a$ ,  $\tau_b$  are the relaxation time of the modes a and b

Choosing the thermodynamics of an ideal gas

$$\left\langle \delta M^{a} \delta M^{b} \right\rangle = m_{aj} m_{bj} \frac{kT}{c_{s}^{2}} n_{j}^{eq} \delta_{ab}$$

**Follows Poisson statistics** 

Equilibration ratio of density, velocity, and stress modes

Without ghost modes



#### **"Immersed Boundary Method" + LBM**

get\_forces.c : hi\_force()

Ahlrichs and Dunweg (1998)



- Couple MD simulation of chains with LBM
- At each time step:
  - (1) LBM calculates the flow field
  - (2) Fluid velocity at bead position is calculated by interpolation
  - (3) Friction force on the monomer is calculated

 $F_f = -\zeta(U_p - U_f(x))$ 

 $\zeta = 6 \pi \eta a$ 

(4) Momentum change is distributed among the nearest lattice neighbors

(5) Fluctuation is added to the monomers to balance frictional losses

$$<\delta F(x,t)\delta F(x',t') >= 2k_B T \zeta \delta(x-x')\delta(t-t')$$

Afternoon session 1. Using lattice Boltzmann :

- A. How to choose the appropriate parameters.
- B. Generate simple shear flow and Poiseuille flow.
- C. Compare with the analytical results and error estimation.
- D. Coupling LB to a point force monomer and validation of fluid field.

Afternoon session 2. Coupling polymer dynamics with the fluid :

- A. How to choose the appropriate parameters for a polymer.
- B. Incorporating thermal fluctuations to polymer and fluid.
- C. Validating hydrodynamic interactions.
- D. Monomers and polymers in Poiseuille flow. (will they migrate ?)



#### Headers

header.h Define limit parameters Include all libraries

lbe.h Define fluid parameters

struct.h Define particle datatype and parameters

macro.h Define macros func\_proto.h Declare global functions [ylchen@polyui2 LBE]\$ make serial

icc -I/usr/local/intel/mkl/10.0.3.020/include -o aslbe main.o driver.o update.o lbe\_update.o bnodes.o bnodes\_init.o implicit\_force.o lbe.o modes\_write.o lub.o velcs\_update.o hs3d.o n\_list.o objects\_init.o objects\_map.o clusters.o cluster\_force.o cluster\_update.o cj\_grad.o global\_sums.o utils.o output.o initlbe.o init\_polymer.o ran\_num.o get\_forces.o verlet\_update.o msg\_ser.o -L/usr/local/intel/mkl/10.0.3.020/lib/32 -lmkl\_ia32 -lpthread ipo: remark #11000: performing multi-file optimizations ipo: remark #11005: generating object file /tmp/ipo\_iccmallY9.o

Intel C compiler Intel math kernel library – vector random number generator

#### Inputs for the polymer parameters

# of chains Total # of monomers Box size	[ylchen@polyui2 LBE]\$ more init/p_inp.dat		
	<pre># Num_chain num_beads max_x max_y max_z</pre>		
	20 220 20 20 20		
	<pre># spring_type(0=FENE,1=WLC) ev_type(0=HS,1=WCA,2=gaussian)</pre>		
	verlet_type(0=1storder,1=2ndorder) initconfig(1=random,2=stretched)		
Force parameters	1 2 2 1		Force type: spring,
	# H_fene Q_fene	kuhn_length nks	EV
	100 3.0 0.212 19.8		Integration
	# evcutoff fric dt	mon_mass kT	Init configuration
Force parameters	5.0 0.483 0.1 1.0 1	0	
MD integration step	# mon_fext		
	0.0 0.0 0.0	Externally applied forces	
	<pre># write_props write_config write_fluid</pre>		
	1000 1000 1000		
	]	Period of output	

```
[ylchen@polyui2 LBE]$ more init/input_file.dat
                                                             # of checkpoints
                             num_modes
                                                             # of steps / checkpoint
 num cycle
               num_step
                                            t_lbe
   1
                   10000
                               0
                                          1.0
                                                             # of output fluid modes
            backflow_flag
                                                             lbe relaxation time
 wall_flag
                             add noise
            0
                                      2
   1
                                                             Wall type: 0 = no walls
  rho
          VX
                 VV
                       VZ
                                                                         1 = \pm v walls
  1.0
          0.0
                0.0
                      0.0
                                Fluid density ratio,
                                                                         2 = +/-yz walls
 mass_fac
              vel_fac
                                velocity ratios
                                                             Backflow
  1.0
             1.0
                           Lubrication cutoff
                                                             Add_noise: 0 = no noise
 lub cut
             del_hy
                           Hi-lubrication factor
                                                                          1 = particle noise
  0.0
            0.001
                                                                         2 = +fluid noise
 sph_fx
             sph_fy
                         sph_fz
                                       External forces on colloids
  0.0
            0.0
                       0.0
 ext fx
             ext_fy
                        ext_fz
                                       External forces on fluid
  0.0
             0.0
                       0.0
 wall_vx
            wall_vy
                          wall_vz
                                       Wall velocities
  0.0
            0.01
                        0.0
  tau
            tau_v
                      tau_g
  1.0
             1.0
                     1.0
                                       Fluid relaxation times for multi-relaxation model
  lub N
              lub T
                         lub R
                        0.43
  0.67
              0.5
                                       Lubrication force parameters
  seed
 2772301
                              Random number seed
```

#### Run

[ylchen@polyui2 LBE]\$ ./aslbe > output [ylchen@polyui2 LBE]\$ more output Running on 1 processors Begin driver: proc # 0, task 0 Adding 20 chains, 11 monomer / chain, 220 particles #chain 0 #monomer 0 at (1.159345e+01 4.784072e+00 1.323395e+01) #monomer 1 at (1.036758e+01 6.397857e+00 1.233972e+01) #monomer 2 at (9.074085e+00 6.012436e+00 1.094426e+01) #monomer 3 at (9.058377e+00 7.348096e+00 1.158538e+01) #monomer 4 at (9.685675e+00 8.463016e+00 1.137528e+01) #monomer 5 at (9.408609e+00 9.148439e+00 1.237802e+01) #monomer 6 at (8.566243e+00 1.033286e+01 1.334280e+01) . . . #monomer 216 at (2.670195e+00 2.151291e-01 2.052470e+00) #monomer 217 at (2.376896e+00 1.898285e+01 3.773407e+00) #monomer 218 at (1.162502e+00 2.566756e-01 5.158859e+00) #monomer 219 at (6.259781e-01 1.303433e+00 6.058158e+00) ncycle 0, numcycle 1 Total mass 2.88000e+05 Fluid mass 0.00000e+00Particle mass 0.00000e+00Map updates 1 Front location 1 Max cluster 0 List updates 13 Collision rate 0.000e+00Max collisions 0.000e+00Total momenta 0.00000e+00 0.00000e+00 0.00000e+00 Begin checkpoint 1 End checkpoint 1 Begin data output 1 on 0End data output 1 on 0End driver: proc #:0, task 0 Elapsed time on proc 0: 1.065000e+01 (0.000000e+00 1.065000e+01)

#### Output

[ylchen@polyui2 LBE]\$ dir data

avg\_disp2.datchain\_props.12.datchain\_props.17.datchain\_props.3.datchain\_props.8.datmonpos.datavg\_props.datchain\_props.13.datchain\_props.18.datchain\_props.4.datchain\_props.9.datp\_out.000chain\_props.0.datchain\_props.14.datchain\_props.19.datchain\_props.5.datchk\_f.000properties.datchain\_props.10.datchain\_props.15.datchain\_props.1.datchain\_props.6.datchk\_p.000run\_time.datchain\_props.11.datchain\_props.16.datchain\_props.2.datchain\_props.7.datfinal.configu.t0000.vtku.t0000.vtklond0.t0000.vtklond0.t0000.vtklond0.t0000.vtklond0.t0000.vtk

avg\_disp2.dat : average monomer MSD, avg velocity avg\_props.dat : average chain MSD, Rg2, stretch chain\_props.dat : chain COM, Rg2, stretch, MSD chk\_f.000: fluid field checkpoint chk\_p.000: chain properties checkpoint final.config: final chain configuration monpos.dat: monomer positions and velocities at different times p\_out.000: colloid properties properties.000: simulation input parameters run\_time.dat: simulation run time bond0\_t????.vtk: paraview configuration files for the polymer u\_t????.vtk: paraview configuration files for the fluid field.

#### **Appropriate Parameters for lattice Boltzmann**

Lattice spacing should be bigger than the solvent molecule mean free path

$$\mathrm{Kn} = l_{\mathrm{mfp}} / \mathrm{dx} << 1$$

Fluid velocity shall not be faster than the speed of momentum propagation (speed of sound)

$$Ma = u / c_s << 1$$

Fluid viscosity must be positive  $\eta/\rho = (t_{coll} - \frac{1}{2}) * c_s^2$ LB code parameters:  $\rho=36$ ,  $\eta=6$ ,  $c_s^2 = 1/3$  $t_{coll} > 1/2$ 

Computational time ~ volume => choose the smallest box possible to obtain correct physics (typically  $L_x$  at least > 8)

Error ~  $O(dx^2)$ 

#### **Exercise 1-2: Generate flow between two parallel plates**

$$\nabla p = \eta \nabla^2 \mathbf{u} + \mathbf{f}$$

$$u(y) = u_{max} \left( \left(\frac{H}{2}\right)^2 - (y - y_{center})^2 \right)$$

What is the error ?

(at the wall)





#### **Exercise 3:** A single monomer (point force) in fluid

The monomer is fixed at a given position, defined in init/init.config



Plot the resulting fluid field using paraview or xmgrace

Compare with the analytical solution in Uofr.dat

What happens if the point force is moved towards the wall ?





#### **Appropriate Parameters for Polymer model**

The distance between monomers should be around 1 lattice spacing

 $\sigma \approx dx$ 

Box size should be > 5 \* Polymer radius of gyration

 $L > 5 R_{g}$ 

Run time should be much longer than the polymer relaxation time to sample equilibrium properties

# **Exercise 5a: A single monomer (point force) in Poiseuille flow**

The monomer is fixed at a given position, defined in init/init.config

Plot the resulting particle trajectory using xmgrace

# **Exercise 5b:** A single dumbbell (point force) in **Poiseuille flow**

A dumbbell is a polymer with only two beads

The dumbbell position can be defined in init/init.config Plot the resulting dumbbell trajectory using xmgrace

Try different initial positions and orientations

# **Exercise 6ab: Trajectories a large number of monomers and dumbbells in Poiseuille flow**

Generate random initial particle positions using

initconfig = 1 in p\_inp.dat

Plot the resulting dumbbell trajectories using xmgrace

tail --lines=100 --quiet data/chain\_props.\*.dat >>
combined\_chainprops.dat