Problem 1. (10 points for each part)

A hypothetical system. Consider a 3-dimensional polymer solution of polymer concentration ϕ. In dilute solution the hypothetical polymer size obeys the scaling law: $R \sim b\, N^{2/5}$. In the concentrated MELT state the polymer is a ROD.

(a) What is the polymer size as a function of concentration, $R(\phi)$, in the semidilute regime ?

(b) The osmotic pressure of the dilute solution is

$$\Pi = kT\phi/N$$

In the melt, the osmotic pressure is independent of N. What is the scaling relation for the osmotic pressure in the semi-dilute regime ?

Problem 2.

Consider a polymer confined in a square channel of height H.

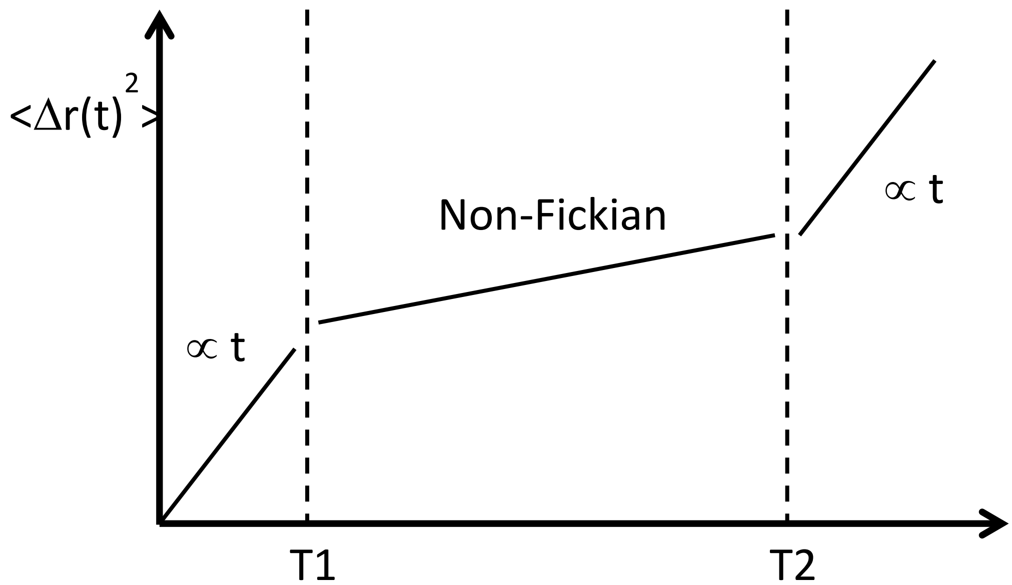
a) How does the polymer radius of gyration depend on H ?

b) With Rouse dynamics (neglecting hydrodynamic interactions), how does the polymer diffusion coefficient depend on H ?

c) With Zimm dynamics (including hydrodynamic interactions) in both unconfined and strongly confined scenarios, how does the polymer diffusion coefficient depend on H ?

3). Consider a very dense hard sphere colloid suspension

a) When the volume fraction is $\phi = 0.54$, the material is still a fluid. The mean square displacement of a single colloid in the suspension is observed to be



Note the two indicated characteristic times T1 and T2

Carefully SKETCH the following and indicate time or frequency scales in terms of T1 and T2

(i) The stress relaxation function G(t)

(ii) On the same plot, and in a LOG-LOG format, plot the elastic modulus $G'(\omega)$ and the loss modulus $G''(\omega)$

b) The volume fraction is now increased to $\phi = 0.56$. The suspension is still a fluid. Do you expect the ratio T2 / T1 to be larger, smaller, or the same as for the $\phi = 0.54$ suspension. Justify your answer in physical terms.

c) The suspension is now concentrated to a volume fraction above the glass transition. Sketch what you expect $G'(\omega)$ now looks like.

4. For a fluctuating particle in an external field, we can consider the simplest external potential as a harmonic oscillator, i.e. $U(r) = k_s r^2$ and $F(r) = -k_s r$.

a) Start with the generalized Langevin equation and derive the velocity autocorrelation function for $<v(0)\, v(t)>$

b) The ratio of the friction coefficient to mass (ζ/m) is the relaxation time of a fluid response to a perturbation. Describe the physical behavior of the velocity autocorrelation function for

i) $(\zeta/m)^2 - 4k_s/m > 0$

ii) $(\zeta/m)^2 - 4k_s/m = 0$

iii) $(\zeta/m)^2 - 4k_s/m < 0$