#2)

$$G''(\omega) = \omega \int_0^\infty dt \cos\omega t \, G(t)$$

$$G'(\omega) = \omega \int_0^\infty dt \sin\omega t \, G(t)$$

$$\boxed{G(t) \equiv G_0 \, e^{-t/\tau_s} / (1 + \sqrt{t/\tau_F})}$$

One CANNOT do integrals analytically

one could do NUMERICALLY, BUT all I wanted was the following ROUGH ANALYSIS

think about different t/w regimes + how G(t) CAN be approximated.

a) 3 regimes:

i) $t \ll \tau_F \Rightarrow G(t) \simeq G_0$ ... constant as solid

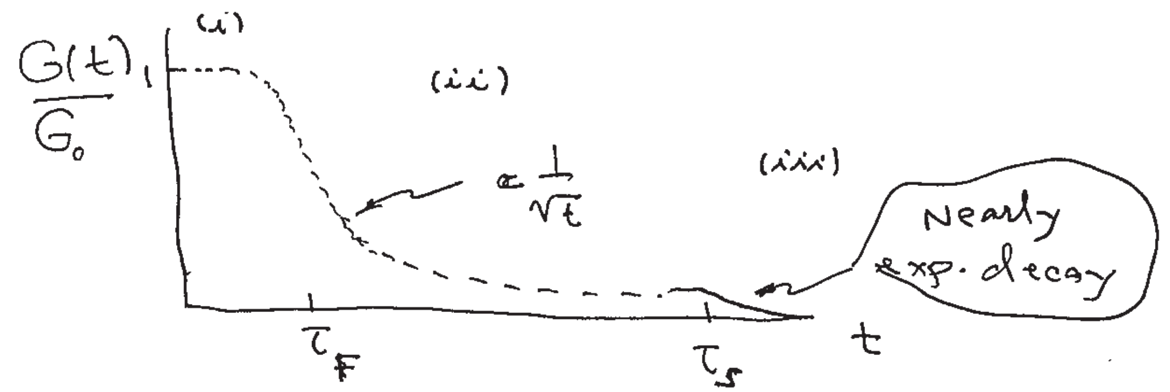
$\omega \tau_F \gg 1$

ii) $\tau_S \ll t \ll \tau_F \Rightarrow G(t) \simeq G_0 \sqrt{\frac{\tau_F}{t}}$

$\frac{\tau_F}{\tau_S} \gg \omega \tau_F \gg 1$ power-law decay

iii) $t \gg \tau_F \Rightarrow G(t) \propto e^{-t/\tau_s}$

.. neglect power Law decay since it is slow compared to exponential.



regime i) & iii) is as MAXWELL model worked out in class. For regime ii)

$$G'(\omega) \cong \omega \int_{\tau_F}^{\tau_S} dt \ \sin \omega t \sqrt{\frac{\tau_F}{t}}\, G_0 \ ; \quad \text{Let } x \equiv t/\tau_F$$

note limits

$$\Rightarrow \ \frac{G'(\omega)}{G_0} \simeq (\omega\tau_F) \int_1^{\tau_S/\tau_F} dx \ \frac{\sin(x(\omega\tau_F))}{\sqrt{x}}$$

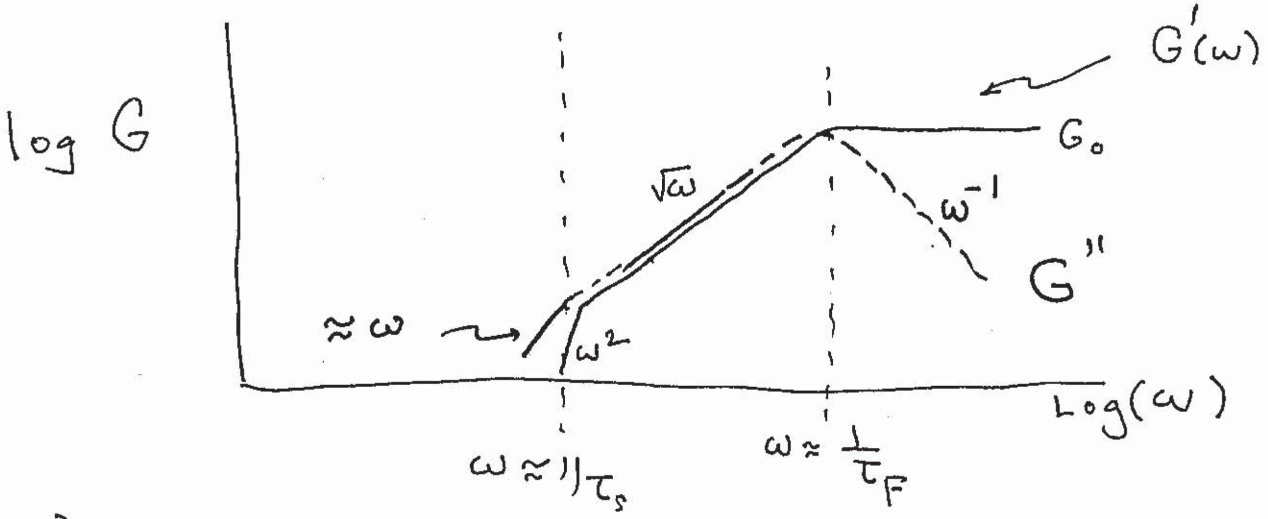
Now, $\tau_S/\tau_F >>> 1$, so approximate as ∞.

Let $y \equiv x(\omega\tau_F) \Rightarrow$ change variables

$$\frac{G'(\omega)}{G_0} = (\omega\tau_F) \underbrace{\left[\int_1^{\infty} dy \ \frac{\sin(y)}{\sqrt{y}}\right]}_{\text{Just a \#}} \frac{1}{\sqrt{\omega\tau_F}} \propto \sqrt{\omega\tau_F}$$

One can similiarly show $\frac{G''}{G_0} \propto \sqrt{\omega\tau_F}$

This $G' \propto G''$ behavior is a consequence of the fractional power law decay of $G(t)$ in regime (ii). Such behavior is exptl'ly seen in colloid & polymers near a gel pt. & short chain polymer liquids.



b) $$\eta = \int_0^\infty dt\, G(t) \underset{\text{3 regimes}}{\approx} G_0 \tau_F + G_0 \int_{\tau_F}^{\tau_s} dt \sqrt{\frac{\tau_F}{t}} + G_0 \int_{\tau_s}^{\infty} dt\, e^{-t/\tau_s}$$

$$= G_0 \tau_F + 2\sqrt{\tau_F}\left\{\sqrt{\tau_s} - \sqrt{\tau_F}\right\} G_0 + G_0 \tau_s \cdot e^{-1}$$

$$= G_0 \tau_s \left[e^{-1} + \underbrace{\frac{\tau_F}{\tau_s} + 2\left(\sqrt{\frac{\tau_F}{\tau_s}}\right.}_{} \left. - \frac{\tau_F}{\tau_s}\right) \right]$$

Negligible since $\frac{\tau_F}{\tau_s} <<< 1$

$$\Rightarrow \boxed{\eta \approx G_0 \tau_s / e}$$

VERY ROUGH

This problem is AN example of the "art" of estimation!